ACTEX Learning

Study Manual for Exam 8

2nd Edition

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A CAS Exam

Actuarial & Financial Risk Resource Materials
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Note: * Not required, but the CAS highly recommends that candidates read these chapters as the candidates will be assumed to be familiar with the material in these chapters.

NOTES

This is my tenth update of the manual. There have been enough syllabus changes and new problems that most of the outline material is mine. For the current update, I reflected the 2023 syllabus material. Effective with the 2020 exams, the Casualty Actuarial Society is no longer providing questions and answers for its exams, so the questions and answers from the 2020 and subsequent exams are not included in this update.

Beginning with the Fall 2021 Study Kit, the state pages that were contained with prior versions of the NCCI *Experience Rating Plan Manual* are no longer provided. The CAS's official response to this for the Fall 2021 Study Kit is on page B579.

Also included in this manual are outlines of Bahnemann Chapters 1-4 that are not required (Chapters 5 and 6 are required). The CAS highly recommends that candidates read Bahnemann Chapters 1-4, as candidates will be assumed to be familiar with the material in these chapters.

When readings have been taken off the syllabus or revised, I retained those questions from past exams that I think are relevant to the current readings, making changes to reflect the then-new syllabus material where appropriate. I also created some questions for some of the then-new material. There are no questions for Bahnemann Chapters 1-4 as they are not required reading on the syllabus. There is a section at the end Section C for Integrative Questions that apply to multiple readings.

Questions and parts of some questions have been taken from material copyrighted by the Casualty Actuarial Society. They are reproduced in this study manual with the permission of the CAS solely to aid students studying for CAS exams. Students may also request past exams directly from the CAS or find them on the CAS website. I am very grateful to the CAS for its cooperation and permission to use this material. The CAS is not responsible for the structure or accuracy of this manual. In some cases, questions and answers have been edited or altered to be more accurate, reflect syllabus changes, or provide a better organized manual. Students should keep in mind that there may be more than one correct way to answer a question even if only one is shown.

Exam questions are identified by numbers in parentheses at the end of each question. Questions have four numbers separated by hyphens. For questions from the exams those numbers represent: the year of the exam, the number of the exam, the number of the question, and the points assigned. Questions I added also have four numbers separated by hyphens and reflect: year I added them, the number of the exam (8), the number of the question I added that year, and the points I assigned. The past exam questions that are retained for which I made significant changes so that they reflect the new syllabus material are designated by: the year of the exam, the number of the exam, the number of the question & MTS, and the points assigned. I did not change the question label for minor changes to reflect the new syllabus material.

My thanks to Peter J. Murzda, Jr., FCAS, ASA, who originally wrote this manual, and Dean A. Westpfahl, FCAS, MAAA, who previously updated it.

I made a conscientious effort to eliminate mistakes and incorrect answers, but a few may remain. I am grateful to students who previously pointed out errors and encourage those who find others to bring them to my attention. If you find any errors, please send them to support@actexmadriver.com. Please check the ACTEX website for corrections subsequent to publication.

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Actuarial Standards Board,
"Actuarial Standard of Practice No. 12, Risk Classification (for All Practice Areas),"
December 2005, Updated for Deviation Language Effective May 1, 2011

OUTLINE

I. SECTION 1. PURPOSE, SCOPE, CROSS REFERENCES, AND EFFECTIVE DATE

A. Purpose – Provides guidance to actuaries when performing professional services with respect to designing, reviewing, or changing risk classification systems.

B. Scope

1. Applies to all actuaries when performing professional services with respect to designing, reviewing, or changing risk classification systems used in connection with financial or personal security systems regarding the classification of individuals or entities into groups intended to reflect the relative likelihood of expected outcomes.

a. Expert testimony
b. Regulatory activities
c. Legislative activities
d. Statements concerning public policy

2. Also applies when giving advice with respect to a risk classification system.

3. Risk classification can affect and be affected by many actuarial activities, such as:

a. Setting rates, contributions, reserves, benefits, dividends, or experience refunds
b. Analysis or projection of quantitative or qualitative experience or results
c. Underwriting actions
d. Development assumptions

4. Standard applies when activities directly or indirectly involve designing, reviewing, or changing a risk classification system.

5. Also applies when performing such activities if those activities directly or indirectly are likely to have a material effect on the intended purpose or expected outcome of the risk classification system.

6. Departures from this standard should be disclosed.

C. Cross References

1. Referenced documents are as amended, restated, or succeeding.

2. If there is a material difference from the originally referenced document, the actuary should consider the guidance in this standard to the extent it is applicable and appropriate.

D. Effective Date – Any professional service commenced on or after May 1, 2006.

II. SECTION 2. DEFINITIONS

A. Advice - An actuary's communication or other work product in oral, written, or electronic form setting forth the actuary's professional opinion or recommendations concerning work that falls within the scope of this standard.

B. Adverse Selection - Actions taken by one party using risk characteristics or other information known to or suspected by that party that cause a financial disadvantage to the financial or personal security system (sometimes referred to as antiselection).

C. Credibility - A measure of the predictive value in a given application that the actuary attaches to a particular body of data (predictive is used here in the statistical sense and not in the sense of predicting the future).

D. Financial or Personal Security System - A private or governmental entity or program that is intended to mitigate the impact of unfavorable outcomes of contingent events. Examples of financial or personal security systems include auto insurance, homeowners insurance, life insurance, and pension plans, where the mitigation primarily takes the form of financial payments; prepaid health plans and continuing care retirement communities, where the mitigation primarily takes the form of direct service to the individual; and other systems, where the mitigation may be a combination of financial payments and direct services.

E. Homogeneity - The degree to which the expected outcomes within a risk class have comparable value.

F. Practical - Realistic in approach, given the purpose, nature, and scope of the assignment and any constraints, including cost and time considerations.

G. Risk(s) - Individuals or entities covered by financial or personal security systems.

H. Risk Characteristics - Measurable or observable factors or characteristics that are used to assign each risk to one of the risk classes of a risk classification system.

I. Risk Class - A set of risks grouped together under a risk classification system.

J. Risk Classification System - A system used to assign risks to groups based upon the expected cost or benefit of the coverage or services provided.

III. SECTION 3. ANALYSIS OF ISSUES AND RECOMMENDED PRACTICES

A. Introduction

1. Approaches to risk classification can vary significantly.

2. It is appropriate for the actuary to exercise considerably professional judgment.

B. Considerations in the Selection of Risk Characteristics

1. Relationship of Risk Characteristics and Expected Outcomes

a. The actuary should select risk characteristics that are related to expected outcomes.

b. A relationship exists if it can be shown that the variation in actual or reasonably anticipated experience correlates to the risk characteristic.

c. To demonstrate a relationship can use:

- Analysis of available data
- Clinical experience
- Expert opinion

d. Rates are considered to be **equitable (fair)** if the differences in rates reflect material differences in expected cost for risk characteristics.

e. The actuary should consider the interdependence of risk characteristics and make appropriate adjustments if their impact on the operation of the risk classification system is expected to be material.

f. It may be appropriate for the actuary to make inferences without specific demonstration. For example, it might not be necessary to demonstrate that persons with seriously impaired, uncorrected vision would represent higher risks as operators of motor vehicles.

2. Causality – It is not necessary to establish a cause and effect relationship between the risk characteristics and expected outcome.

3. Objectivity – The actuary should select risk characteristics that are capable of being objectively determined.

4. Practicality – The actuary's selection of a risk characteristic should reflect the tradeoffs between practical and other relevant considerations such as:

   a. Cost, time, and effort needed to evaluate the risk characteristic

   b. The ongoing cost of administration

   c. The acceptability of the usage of the characteristic

   d. Potential usage of different characteristics that would produce equivalent results

5. Applicable Law – The actuary should consider whether compliance with applicable law creates significant limitations on the choice of risk characteristics.

6. Industry Practices – The actuary should consider usual and customary risk classification practices for the type of financial or personal security system under consideration.

7. Business Practices – The actuary should consider limitations created by business practices related to the financial or personal security system as known to the actuary and consider whether such limitations are likely to have a significant impact on the risk classification system.

C. Considerations in Establishing Risk Classes

1. Intended Use

   a. A risk classification system should be appropriate for the intended use.

   b. Different sets of risk classes may be appropriate for different purposes.

2. Actuarial Considerations

   a. Adverse Selection

      - Likely to occur if the variation in expected outcomes within a risk class is too great.

      - To the extent practical, the actuary should establish risk classes such that each has sufficient homogeneity with respect to the expected outcomes to satisfy the purpose for which the risk classification system is intended.

   b. Credibility

      - It is desirable that risk classes be large enough to allow credible statistical inferences regarding expected outcomes.

      - When this is not possible, the actuary should balance considerations of predictability with considerations of homogeneity.

      - The actuary should use professional judgement to achieve this balance.

3. Other Considerations – The actuary should:

   a. comply with the applicable law;

   b. consider industry practices for that type of financial or personal security system as known to the actuary; and

   c. consider limitations created by business practices of the financial or personal security system as known to the actuary.

4. Reasonableness of results – The actuary should consider the reasonableness of results that proceed from the intended use of the risk classes such as:

   a. consistency of the patterns of rates;

   b. consistency of values; and

   c. consistency of factors among risk classes.

D. Testing the Risk Classification System

1. Upon the establishment of the risk classification system and upon subsequent review, the actuary should, if appropriate, test the long-term viability of the financial or personal security system.

2. When performing such tests subsequent to the establishment of the risk classification system, the actuary should evaluate emerging experience and determine whether there is any significant need for change.

3. Effect of Adverse Selection

   a. Can potentially threaten the long-term viability of a financial or personal security system.

   b. If the effects of adverse selection are expected to be material, the actuary should, when practical, estimate the potential impact and recommend appropriate measures to mitigate the risk.

4. Risk Classes Used for Testing

   a. The actuary should consider using a different set of risk classes for testing long-term viability than was used as the basis for determining the assigned values.

   b. This is likely to improve the meaningfulness of the tests.

5. Effect of Changes – The actuary should consider testing the effects of changes:

   a. if the risk classification system has changed; or

   b. business or industry practices have changed.

6. Quantitative Analyses – Depending on the purpose, nature, and scope of the assignment, the actuary should consider performing quantitative analyses of the impact of the following to the extent they are generally known and reasonably available to the actuary:

   a. significant limitations due to compliance with applicable law;

   b. significant departures from industry practices;

   c. significant limitations created by business practices of the financial or personal security system;

   d. any changes in the risk classes or the assigned values based upon the actuary's determination that experience indicates a significant need for a change; and

   e. any expected material effects of adverse selection.

E. Reliance on Data or Other Information Supplied by Others – Refer to ASOP No. 23, *Data Quality*.

F. Documentation

1. The actuary should document the assumptions and methodologies used in designing, reviewing, or changing a risk classification system in compliance with the requirements of ASOP No. 41, *Actuarial Communications*.

2. The actuary should also prepare and retain documentation to demonstrate compliance with the disclosure requirements in Section 4.1 of this standard.

IV. SECTION 4. COMMUNICATIONS AND DISCLOSURES

A. Communications and Disclosures (Section 4.1)

1. When issuing actuarial communications under this standard, the actuary should comply with ASOP Nos. 23 and 41.

2. In addition, the actuarial communications should disclose any known significant impact resulting from the following to the extent they are generally known and reasonably available to the actuary:

   a. significant limitations due to compliance with applicable law;

   b. significant departures from industry practices;

   c. significant limitations created by business practices related to the financial or personal security system;

   d. a determination by the actuary that experience indicates a significant need or change, such as changes in the risk classes or the assigned values; and

   e. expected material effects of adverse selection;

   f. the disclosure in ASOP No. 41, section 4.2, if any material assumption or method was prescribed by applicable law (statutes, regulations, and other legally binding authority);

   g. the disclosure in ASOP No. 41, section 4.3, if the actuary states reliance on other sources and thereby disclaims responsibility for any material assumption or method selected by a party other than the actuary; and

   h. the disclosure in ASOP No. 41, section 4.4, if, in the actuary's professional judgment, the actuary has otherwise deviated materially from the guidance of this ASOP.

3. The actuarial communications should also disclose any recommendations developed by the actuary to mitigate the potential impact of adverse selection.

PAST CAS EXAMINATION AND NEW QUESTIONS

1. The Supreme Court's decision in the *Norris* case eliminated the use of sex as a rating variable in pensions. Discuss the potential implications of this decision on automobile insurance classification in the context of the considerations in the selection of risk characteristics discussed in ASOP 12.

   (84–9–9 & MTS–3)

2. A property insurance company is considering adding a new classification rating variable to its homeowners insurance program based on an individual risk's actual loss experience over the past five-year period as follows:

   Class A – no claims Class B – one or two claims Class C – three or more claims

   Considering the considerations in the selection of risk characteristics discussed in ASOP 12, would you recommend the addition of this new classification? Why or why not?

   (96–9–48b & MTS–1.5)

3. As the personal lines actuary for the department of insurance in the state of Crazyfornia, you have been asked by the state's insurance commissioner to comment on Proposition 99.

   Proposition 99 – The ratemaking for personal automobile insurance should be based on a new classification system using the following six criteria:

   a) Insureds are to be classified based on nationality.
   b) Insureds are to be classified based on the ability to pass an annual random drug test.
   c) Insureds are to be classified based on whether they can pass a comprehensive, individually administered eight-hour driving test every year.
   d) Insureds are to be classified based on their weights.
   e) Insureds are to be classified as either good eyesight or bad eyesight. Each eye doctor can have his/her own definition of good/bad eyesight.
   f) Insureds are to be classified as right-handed or left-handed.

   For each criterion, identify which one of the considerations in the selection of risk characteristics discussed in ASOP 12 is violated. You may not use the same consideration for more than two criteria.

   (97–9–48 & MTS–0.5/0.5/0.5/0.5/0.5/0.5)

1. ASOP 12 lists the following considerations in the selection of risk characteristics:

   1) There should be a relationship between the risk characteristics and the expected outcome.
   2) The risk characteristics should be objective.
   3) The risk characteristics should reflect the tradeoff between practicality and other considerations.
   4) Risk characteristics should comply with applicable law.
   5) The actuary should consider industry practices in selecting risk characteristics.
   6) The actuary should consider business practices in selecting risk characteristics.

   If the *Norris* case were applied to automobile insurance, it would violate 1) because the risk classification system would not reflect expected costs as males would be undercharged and females overcharged as one relevant cost-related factor would be disregarded. The resulting risk classification would meet considerations 2), 3), and 4). It would not meet 5) or 6).

2. See #1 for the list of considerations.

   1) The proposed system would not reflect cost differences among risks or distinguish among risks on cost-based factors because past loss experience may not be a good indicator of future loss experience.

   2) The system could be applied objectively.

   3) The new system would entail extra costs without comparable benefits. In addition, public acceptability would be questionable, given the random penalties the system would produce.

   4) Whether the new system complies with the applicable law would have to be determined for each state.

   5) The new system is not current industry practice, which could be an advantage or disadvantage.

   6) The new system is not current business practice and would add expense.

   On balance, I would not recommend the new classification.

3. See #1 for the list of considerations.

   a. Need one of these. It is not objective (2). It does not comply with applicable law (4).

   b. Need one of these. There may not be a relationship between the risk characteristic and the expected outcome (1). There is not practical due to the added expense (3). It may not comply with applicable law; this would have to be determined for each state (4).

   c. It is not practical due to the added expense (3).

   d. There probably is not a relationship between the risk characteristic and the expected outcome (1).

   e. It is not objective (2).

   f. There is no relationship between the risk characteristic and the expected outcome (1).

4. You are the actuary for Aggressive Mutual Insurance Company. The marketing department has approached you with a plan to increase business by liberalizing protection class definitions. The new definition would allow you to classify any risk within eight miles of the nearest fire department using the protection class of that town, without any verification of its ability to respond to the location of that risk.

   a. Discuss this based on the relationship of risk characteristics and expected outcomes discussed in ASOP 12?

   b. Based this discussion, what would you tell the marketing director about the appropriateness of the proposed class definitions?

   (99–9–43 & MTS–0.75/0.25)

5. Adverse selection is a financial threat to an insurance program's solvency. Based on ASOP 12, answer the following.

   a. Briefly describe adverse selection.
   b. Briefly explain when it is likely to occur in a risk classification system.
   c. Briefly explain how to control it through the risk classification system.

   (00–9–35 & MTS–0.5/0.5/0.5)

6. Which of the following describes a risk classification system that complies with the recommended practices in ASOP 12? For each one that does not apply, make the appropriate correction or comment.

   a. The system should be applied subjectively.
   b. The system should produce prices based on the observed actual losses of each risk.
   c. The system should reflect expected cost differences.
   d. The system should be based solely on complying with the law.
   e. The system should be the same for all competitors.

   (02–9–20 & MTS –0.25/0.25/0.25/0.25/0.25)

4. a. 1) The risk characteristics do not relate to the expected outcomes since those having a lower risk because they live next to a fire department are charged the same rate as those who a higher risk living almost eight miles away.

   2) This system is not fair since those having a lower risk are charged the same as those having a higher risk.

   b. I would tell the marketing director this is not a good idea and recommend it not be carried out.

5. a. Adverse selection is the actions taken by one party using risk characteristics or other information known to or suspected by that party that cause a financial disadvantage to the financial or personal security system.

   b. Adverse selection is likely to occur if the variation in expected outcomes within a risk class is too great.

   c. To the extent practical, the actuary should establish risk classes such that each has sufficient homogeneity with respect to the expected outcomes to satisfy the purpose for which the risk classification system is intended.

6. a. No – Substitute “objectively” for “subjectively.”

   b. No – Substitute “expected” for “observed actual.”

   c. Yes.

   d. No –Complying with the law is only one of several factors.

   e. No – This is not mentioned.

7. Your company is planning to purchase a block of boatowners insurance business from Zeron. Zeron has raised overall rates on this block of business for three consecutive years, but does not classify risks by age or size. Despite the rate increases, loss ratios continue to worsen and growth remains high.

   a. Explain how adverse selection could be impacting the seller's poor results.

   b. Using the information below, calculate rates to address the adverse selection problem. Briefly justify your methods in light of the considerations in establishing risk classes discussed in ASOP 12.

Age Group	Boat Size	Ethnicity Group	Exposures	Premium	Losses
1	Large	A	75	15,000	4,600
1	Medium	A	35	7,000	3,200
1	Small	A	5	1,000	350
1	Large	B	15	3,000	1,100
1	Medium	B	20	4,000	1,800
1	Small	B	45	9,000	6,500
2	Large	A	100	20,000	11,000
2	Medium	A	60	12,000	8,500
2	Small	A	20	4,000	2,500
2	Large	B	25	5,000	2,600
2	Medium	B	25	5,000	2,800
2	Small	B	50	10,000	7,200

(02–9–48 & MTS–1/3)

7. a.

Age Group	Boat Size	Ethnicity Group	Loss Ratio
1	Large	A	0.307
1	Medium	A	0.457
1	Small	A	0.350
1	Large	B	0.367
1	Medium	B	0.450
1	Small	B	0.722
2	Large	A	0.550
2	Medium	A	0.708
2	Small	A	0.625
2	Large	B	0.520
2	Medium	B	0.560
2	Small	B	0.720

Although loss ratios are lower for age group 1, larger boats, and in most cases ethnicity group A, Zeron charges them all the same rate. If a competitor of Zeron has a classification system that separates insureds based on these variables and charges appropriate premiums, insureds in low-risk groups are likely to switch to the competitor since they will receive a lower rate. As this happens, Zeron's book comes to be composed more and more of high-risk insureds and until its rates reflect this greater proportion of such insureds, it will have a higher loss ratio. If the switching to a competitor is gradual, what will happen is a continuing series of rate increases as each rate increase spurs other lower-than-average risk insureds to leave, creating the need for another rate increase.

b. Since using ethnicity as a variable is probably illegal and could create a business public perception problem if not, separate insureds by age group and size to calculate proposed premiums.

1) Calculate relativities for each combination of age group and boat size:

Average Loss Ratio = (Total Losses)/(Total Premiums) = 52,150/95,000 = .549

Age group 1	Large boats	5,700/18,000 = .317	.317/.549 = .577
	Medium boats	5,000/11,000 = .455	.455/.549 = .829
	Small boats	6,850/10,000 = .685	.685/.549 = 1.248
Age group 2	Large boats	13,600/25,000 = .544	.544/.549 = .991
	Medium boats	11,300/17,000 = .665	.665/.549 = 1.211
	Small boats	9,700/14,000 = .693	.693/.549 = 1.262

2) Calculate the indicated premium:

Average Premium = (Total Premiums)/(Total Exposures) = 95,000/475 = 200

Age group 1	Large boats	(200)(.577) = 115.40
	Medium boats	(200)(.829) = 165.80
	Small boats	(200)(1.248) = 249.60
Age group 2	Large boats	(200)(.991) = 198.20
	Medium boats	(200)(1.211) = 242.20
	Small boats	(200)(1.262) = 252.40

8. List five considerations in establishing risk classes mentioned in ASOP 12.

(17-8-MTS-1.25)

9. a. According to ASOP 12, what two items should an actuary balance regarding credibility?

b. How should an actuary accomplish this balance?

(17-8-MTS-0.5/0.5)

10. List four considerations in selecting risk characteristics mentioned in ASOP 12.

(17-8-MTS-1.0)

11. Discuss casualty as a risk characteristic based on ASOP 12.

(17-8-MTS – 1.0)

8. Any five of these are acceptable.

The considerations in establishing risk classes in ASOP 12 are:

1. A risk classification system should be appropriate for the intended use.

2. To the extent practical, the risk classes should have sufficient homogeneity with respect to the expected outcomes to satisfy the purpose for which the risk classification system is intended.

3. It is desirable that the risk classes be large enough to allow credible statistical inferences regarding expected outcomes.

4. The risk classes should comply with applicable law.

5. The actuary should consider industry practices in determining the risk classes.

6. The actuary should consider business practices in determining the risk classes.

7. The risk classes should produce reasonable results that proceed from the intended use of the risk classes.

9. a. An actuary should balance considerations of predictability with considerations of homogeneity.

b. An actuary should use professional judgment in achieving this balance.

10. Any four of these are acceptable.

1) There should be a relationship between the risk characteristics and the expected outcome.
2) The risk characteristics should be objective.
3) The risk characteristics should reflect the tradeoff between practicality and other considerations.
4) Risk characteristics should comply with applicable law.
5) The actuary should consider industry practices in selecting risk characteristics.
6) The actuary should consider business practices in selecting risk characteristics.

11. While the actuary should select risk characteristics that are related to expected outcomes, it is not necessary for the actuary to establish a cause and effect relationship between the risk characteristics and expected outcome in order to use a specific risk characteristic.

12. a. You are given the following information:

Type of Vehicle	Earned Exposures	Number of Claims per Year	Pure Premium
Cars	100,000	5,000	$200
Trucks	75,000	4,000	300

Would a classification plan that assigns cars and trucks to different classes make sense based on ASOP 12 requirements? Explain why or why not.

b. You are given the following information:

Type of Vehicle	Earned Exposures	Number of Claims per Year	Pure Premium
Type A	99,950	4,950	$199
Type B	50	5	2,199

Would a classification plan that assigns type A and type B cars make sense based on ASOP 12 requirements? Explain why or why not

(04–9–23 & MTS–1.5/1.5)

13. A company is considering changing its age-of-home rating system, which has been in use for five years, and has compiled the following data:

	Current	2005–7 Combined			2007
Age of Home	Age Discount	Earned Exposures	Earned Premium ($)*	Loss Ratio	Loss Ratio
0	5%	40,000	28,000,000	54%	27%
1	5%	35,000	23,625,000	65%	62%
2	5%	35,000	23,100,000	65%	50%
3	3%	25,000	16,125,000	60%	48%
4	3%	20,000	12,600,000	45%	40%
5	3%	25,000	15,375,000	60%	53%
6+	0%	30,000	18,000,000	60%	59%
Total		210,000	136,825,000	63%	50%

*At current discounts

Provide a recommendation whether the company should adopt each of the three changes below. Defend the recommendation on the basis of ASOP 12.

a. Set the discount for age 0 (new homes) to 15%, leaving other discounts unchanged.

b. Set the discount for age 4 to 25%, leaving other discounts unchanged.

c. Disaggregate the age 6+ group and implement discounts of 2% for age 6 and age 7 and 1% for age 8 and age 9, leaving discounts for age 10+ at 0%

(08–9–2 & MTS–1/1/1)

12. a. Frequency for cars = 5,000 / 100,000 = 0.050
Frequency for trucks = 4,000 / 75,000 = 0.053

Although the frequencies are similar, the pure premiums are very different. Each group has enough vehicles to allow credible statistical inferences regarding expected outcomes. Therefore, it makes sense put cars and trucks in different rating classes.

b. Frequency for Type A cars = 4,950 / 99,950 = 0.050
Frequency for Type B cars = 5 / 50 = 0.100

Severity for Type A cars = 199 / 0.050 = 3,980
Severity for Type B cars = 2,199 / 0.100 = 21,990

Both frequency and severity are different, but Type B does not have enough vehicles to allow credible statistical inferences regarding expected outcomes. Therefore, it does not make sense to put Type A and Type B cars in different rating classes.

13. a. Premium without discount = 28,000,000 / 0.95 = 29,473,684
Three-year losses = 28,000,000 x 0.54 = 15,120,000
Premium with proposed discount = 29,473,684 x 0.85 = 25,052,631
Three-year loss ratio with proposed discount = 15,120,000 / 25,052,631 = 60%

It should adopt the discount. The classification is the one with the largest number of earned exposures, has sufficient credibility, and has a three-year loss ratio that brings the loss ratio with the proposed premium in line with those of the other rating classes.

b. Premium without discount = 12,600,000 / 0.95 = 13,263,158
Three-year losses = 12,600,000 x 0.45 = 5,670,000
Premium with proposed discount = 13,263,158 x 0.75 = 9,947,368
Three-year loss ratio with proposed discount = 5,670,000 / 9,947,368 = 57%

It should not adopt the discount. Even though the three-year loss ratio with the proposed premium would be in line with those of most of the other rating classes, this is the classification with the fewest earned exposures. There also would be extreme premium discontinuities with ages 3 and 5 that would be problematic to explain to policy holders as their houses age.

c. The data presented are insufficient to make a recommendation regarding the proposed discount. Splitting one classification into the several proposed should increase homogeneity, but the number of exposures in the three new groups might be too small to be credible.

14. List three items that the actuary should consider that proceed from the intended use of the risk classes in terms of whether or not the results are reasonable based on ASOP 12.

(17-8-MTS-0.75)

15. See Robertson 5c.

16. An insurance company is launching a new telematics program for their private passenger automobile book of business. Telematics devices record various attributes such as miles driven and braking practices. Management decided to give a 5% discount to all customers that participate in the program. The Department of Insurance questions the filing and wants the company to address the following potential concerns:

- Risk of adverse selection
- Relationship between risk and expected outcomes
- Practicality of monitoring the discount's effectiveness

Defend the use of the discount by briefly addressing each of the concerns in light of Actuarial Standard of Practice No. 12, Risk Classification.

(18-8-4-0.75)

14. The actuary should considerer:

    1) the consistency of the pattern of rates;
    2) the consistency of values; and
    3) the consistency of factors among risk classes.

15. See Robertson 5c.

16. Risk of adverse selection:
    - Drivers who know that they drive poorly are unlikely to submit to monitoring.

    Relationship between risk and expected outcomes:
    - Drivers are more likely to drive safely if they know they are being monitored. Therefore, drivers with the discount have a lower expected loss cost.

    Practicality of monitoring the discount's effectiveness:
    - Adoption rates and the experience of non-adopters vs. adopters can be analyzed over time.

Robert A. Bailey and LeRoy J. Simon,
"An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car";
Discussion by W. J. Hazam,
***PCAS* XLVI, 1959, pp. 159–64; XLVII, 1960, pp. 150–52**

OUTLINE

I. INTRODUCTION

A. The Problem

What credibility should be assigned to the accident experience of an automobile in determining its liability premium?

B. The Authors' Solution

Automobiles are divided by class, reflecting use and driver characteristics, and by subclass, reflecting claim experience over three years. Their claim frequency per $1,000 of premium for two subsequent years was then compiled and each class's frequency was compared to the average. Credibility equals the ratio of the future relative frequency less unity to the past relative frequency less unity.

C. Symbols

1. A - subclass with three or more accident-free years
2. B - subclass with no accident-free years
3. m - claim frequency of a class
4. N - radix, number of persons in the population
5. R - ratio of actual to expected losses
6. X - subclass with only two accident-free years
7. Y - subclass with only one accident-free year
8. Z - credibility

II. THE AUTHORS' APPROACH

A. Determination of Subclass Relative Frequency

1. Convert each subclass's earned premium to earned premium at rates for subclass B
2. Divide the earned premium by $1,000
3. Divide the number of claims by the premium in 2.
4. Take the ratio of subclass frequency to the average frequency for all classes
5. Compute the relative frequency for the following classes:
   a. A
   b. (A + X)
   c. (A + X + Y)

B. Determination of Accident-Free Credibilities

1. One-year credibility equals unity minus the relative frequency for (A + X + Y)
2. Two-year credibility equals unity minus the relative frequency for (A + X)
3. Three-year credibility equals unity minus the relative frequency for A

C. Determination of Credibility for Risks Having at Least One Accident in the Last Year

1. Calculate past relative frequency for these risks
   a. Assume Poisson applies with average frequency m
   b. Determine the percentage of persons with at least one claim, unity minus the percentage of persons with no claims, i.e., $(1 - e^{-m})$
   c. Average frequency for those with at least one claim

$$AF = \frac{m}{1 - e^{-m}}$$

   d. Substitute actual frequency for m
   e. Relative frequency

$$RF = \frac{1}{1 - e^{-m}}$$

2. Determine the future relative frequency for this group of risks
3. Apply the following formula:

$$Z = \frac{\text{Future Relative Frequency} - 1.0}{\text{Past Relative Frequency} - 1.0}$$

III. PERSPECTIVE

A. The Authors' Major Conclusions for Canadian Private Passenger Cars

1. One-year experience has significant and measurable credibility for experience rating, ranging from .038 to .071 for a subclass
2. In a highly refined private passenger rating system reflecting inherent hazard, the accuracy of a merit rating plan would be low, but if there is a wider range of hazard, credibility would be larger
3. Adding a second year's experience to one year's will increase credibility by two-fifths; adding a third year's experience to two years' will increase credibility by one-sixth

B. Other Comments

1. Class 1 (no male operator under 25) is the least homogeneous and thus its subclasses have the most credibility
2. Credibility also increases with size

C. Reasons That Credibility Does Not Vary in Proportion to Time

1. An individual's accident propensity changes over time
2. The population of a class changes as individuals enter and leave
3. Individuals within a class have different accident propensities, which are markedly skewed
4. In the credibility formula, Z is not exactly proportionate to n

D. Use of Premium as a Base Rather Than Car-Years

1. According to the authors, this avoids the maldistribution created by having territories with higher claim frequencies produce more X, Y, and B risks and higher territorial premiums
2. According to Hazam, this eliminates maldistribution only if both of the following two conditions are met:
   a. High-frequency territories are also high-premium territories
   b. Territorial differentials are proper

E. Use of Losses Rather Than Claim Counts

1. Accident frequency used to reduce chance variations caused by claim size variations
2. But subclass B risks have a consistently higher-than-average severity and subclass A risks have one that is lower than average
3. This consistency is the reason that using losses instead of counts produces an increase in credibility

F. Hazam's Conclusions

1. Credibility is measurable and significant
2. But not large enough to justify the credits now offered by many U.S. plans
3. May, however, reduce the gap by also taking into account conviction frequencies

G. Comparison with Dropkin

1. Use of accidents rather than violations
2. Emphasis on the results accomplished rather than on the limits of segregation
3. Their levels of data
    a. Possible levels of data
        1) Class
        2) Subdivision of class by violations or accidents
        3) Individual drivers
    b. Bailey and Simon
        1) Weight class and subclass data to get a better predictor of future experience
        2) Subclass has some credibility and thus is somewhat homogeneous
        3) Refer to subclass data as individual experience
    c. Dropkin
        1) Separates class data into subclasses
        2) Still finds subclasses heterogeneous and overlapping
        3) Implies that further segregation needed so that subclass experience approximates that of the individual

PAST CAS EXAMINATION QUESTIONS

A. The Credibility Equation

A1. Bailey and Simon in their paper, "An Actuarial Note on the Credibility of a Single Private Passenger Car" computed credibilities for accident-free risks based upon the commonly used experience rating formula, Modification = Z(R) + (1 − Z) where Z is the credibility factor and R is the ratio of actual losses to expected losses. Given the following information and using Bailey and Simon's technique, compute the credibilities for automobiles with at least one, two, and three years, respectively of accident-free driving.

Years Since Last Accident	Earned Car Years	Earned Premium at Present Rates ($000)	Number of Claims Incurred	Claim Frequency per $1,000 of Premiums	Relative Claim Frequency
3	2,757	159,108	217,151	1.365	.920
2	131	7,910	13,792	1.744	1.175
1	164	9,862	19,346	1.962	1.322
None	274	17,226	37.730	2.190	1.476
Total	3,326	194,106	288,019	1.484	1.000

(75S–9a–2a–6)

A2. You have been asked to develop a compensation experience rating plan for one-employee manufacturing risks. For ease of administration, it has been decided that only one year's claim experience will be utilized. The table below gives the last calendar year's experience sorted by the time elapsed since the previous claim.

Years Since Latest Claim	Earned Employee Years	Number of Incurred Claims
0	25,000	7,500
1	75,000	7,500
2 or more	300,000	15,000
Total	400,000	30,000

Determine the amount of credibility that can be assigned to one year's claim experience. (77–9–13–5)

A3. a. From the information below on a particular class of private passenger automobile business, determine the experience rating credibility of the experience of one private passenger car for one year.

Years Since Most Recent Accident	Earned Car Years	Number of Claims	Claim Frequency
0	50,000	9,000	.180
1	50,000	7,500	.150
2	50,000	6,500	.130
3	50,000	6,000	.120
4	800,000	56,000	.070
Total	1,000,000	85,000	.085

b. Assuming the Poisson distribution represents the risk distribution and using the additional information and notations below, set up the final equation you would use to confirm your answer above for the credibility of one year's experience.

N - total number of cars insured x - claim frequency of class

Ne^{-x} - number of cars having no claim last year. (78–9–8–4/4)

A1. 1) Calculate future absolute claim frequencies:

FACF = (Number of Claims Incurred)/(Earned Premium at Present Rates)

$$FACF_1 = \frac{217{,}151 + 13{,}792 + 19{,}346}{159{,}108 + 7{,}910 + 9{,}862} = 1.415$$

$$FACF_2 = \frac{217{,}151 + 13{,}792}{159{,}108 + 7{,}910} = 1.383 \qquad FACF_3 = \frac{217{,}151}{159{,}108} = 1.365$$

2) Calculate future relative claim frequencies:

$FRCF = FACF/FACF_{Overall}$ $\quad FRCF_1 = 1.415/1.484 = .953$

$FRCF_2 = 1.383/1.484 = .932$ $\quad FRCF_3 = 1.365/1.484 = .920$

3) Calculate credibilities:

$Z = 1 - FRCF$ $\quad Z_1 = 1 - .953 = .047$

$Z_2 = 1 - .932 = .068$ $\quad Z_3 = 1 - .920 = .080$, pp. 159–60.

A2. 1) Calculate future absolute claim frequencies:

$$FACF_1 = \frac{\text{Number of Claims Incurred}}{\text{Earned Employee Years}} = \frac{7{,}500 + 15{,}000}{75{,}000 + 300{,}000} = .060$$

$FACF_{Overall} = 30{,}000/400{,}000 = .075$

2) Calculate the future relative claim frequency:

$FRCF_1 = FACF_1/FACF_{Overall} = .060/.075 = .800$

3) Calculate the credibility:

$Z = 1 - FRCF_1 = 1 - .800 = .200$, pp. 159–60.

A3. a. 1) Calculate the future absolute claim frequency:

FACF = (Number of Claims)/(Earned Car Years)

$$FACF_1 = \frac{7{,}500 + 6{,}500 + 6{,}000 + 56{,}000}{50{,}000 + 50{,}000 + 50{,}000 + 800{,}000} = .080$$

2) Calculate the future relative claim frequency:

$FRCF_1 = FACF_1/FACF_{Overall} = .080/.085 = .941$

3) Calculate the credibility:

$Z_1 = 1 - FRCF_1 = 1 - .941 = .059$, pp. 159–60.

b. 1) Calculate the future relative claim frequency:

$FRCF_0 = FACF_0/FACF_{Overall} = .180/.085 = 2.118$

2) Since the past relative claim frequency equals $1/(1 - e^{-x})$ or $1/(1 - e^{-.085})$, we get the following equation:

$$FRCF_0 = 2.118 = \frac{Z_0}{1 - e^{-.085}} + (1 - Z_0) \qquad Z_0 = \frac{(1.118)(1 - e^{-.085})}{e^{-.085}} = .099, \text{ pp. 159–160, 164.}$$

A4. Assume you are reviewing a private passenger automobile rating plan in which the standard rate is $500 and the merit rating plan is based entirely upon claim frequency, as in Bailey and Simon. Based upon the following data, what is the rate for a driver who has had no claims for at least one year?

	Experience for the Following Year	
Number of Years Since Last Accident	Earned Premium at Present Rates	Number of Claims
Two or more	$90,000	99
One	5,000	7
None	5,000	9

(80–9–6–3)

A5. A simplified merit rating plan is maintained that splits insureds into two groups based on accident record in the past year. The two groups are group A, whose insureds have had no accidents and group B, whose insureds have had at least one accident in the past year. The data of the groups are summarized as follows:

	Number of Insureds	Accidents in Prior Year	Subsequent Pure Premium
Group A	900	0	$50
Group B	100	150	$60

Using Bailey and Simon's approach and assuming no distributional biases, calculate the credibility of the experience of a single group B insured. (81–9–21–3)

A6. East Dakota requires a merit rating plan for workers compensation risks not eligible for experience rating. The modification is based on the latest year's loss ratio developed to ultimate. In a recent year, the loss-free non-experience-rated risks of the previous year developed a 58.5% loss ratio, while the non-experience-rated risks as a whole duplicated the 60.0% permissible. Calculate the credibility using the method of Bailey and Simon. (83–9–6–2)

A7. A simplified merit rating plan splits a class of insureds into two groups based on accident record in the past year. The members of group I have had no accidents and the members of group II have had at least one accident in the past year. The data of the groups are summarized as follows:

	Number of Insureds	Accidents in Prior Year	Subsequent Pure Premium
Group I	800	0	$250
Group II	200	250	$300

Using Bailey and Simon's approach and assuming no distributional biases, calculate the credibility of the experience of a single group II insured. (84–9–6–3)

A8. Bailey developed a formula for expected claim frequency for risks with n or more accident free years as $r/(a + n)$. Using the Bailey and Simon approach, calculate the credibility of the experience of risks that are accident free for five or more years, given $r = 10$ and $a = 100$. (85–9–13–2)

A9. You are evaluating the experience rating credibilities for a book of private passenger auto experience. You have decided that the Poisson distribution is a reasonable model for the claim frequency distribution. Furthermore, you observe that the claim frequency is m. Moreover, risks classified as 1B, which have had one or more claims in the past year, are observed to have a subsequent claim frequency of $m(e^{-m} + 1)$. Give an expression in terms of m for the credibility of 1B risks for a one-year experience period. (86–9–9–3)

A4. 1) Calculate the future absolute claim frequencies:

$$FACF_1 = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = \frac{99+7}{90{,}000+5{,}000} = .001116$$

$FACF_{Overall} = (99 + \& + 9)/(90{,}000 + 5{,}000 + 5{,}000) = .00115$

2) Calculate the future relative claim frequency:
$FRCF_1 = FACF_1/FACF_{Overall} = .001116/.00115 = .970$

3) Calculate the credibility: $Z_1 = 1 - FRCF_1 = 1 - .970 = .030$

4) Calculate the premium:
$Premium_1 = (\text{Base Rate})(1 - Z_1) = (500)(1 - .03) = 485$, pp. 159–60.

A5. 1) Calculate the past absolute accident frequencies:
PAAF = (Number of Accidents)/(Number of Insureds)
$PAAF_B = 150/100 = 1.500$ $\quad PAAF_{Overall} = 150/(900 + 100) = .150$

2) Calculate the past relative claim frequency:
$PRAF_B = PAAF_B/PAAF_{Overall} = 1.50/.15 = 10.0$

3) Calculate the future relative accident frequency, assuming it varies directly with pure premium.

$$FRAF_B = \frac{\text{Future Pure Premium}_B}{\text{Future Pure Premium}_{Overall}} = \frac{60}{(.90)(50) + (.10)(60)} = 1.176$$

4) Calculate the credibility:

$$Z_B = \frac{FRAF_B - 1.0}{PRAF_B - 1.0} = \frac{1.176 - 1.000}{10.0 - 1.0} = .020, \text{ pp. 159–60.}$$

A6. 1) Calculate the relative loss ratio:
RLR = (Loss Ratio for Loss-Free Group)/(Overall Loss Ratio) = .585/.600 = .975

2) Calculate the credibility: $Z = 1 - RLR = 1 - .975 = .025$, pp. 159–60.

A7. 1) Calculate past absolute accident frequencies:

$$PAAF_{II} = \frac{\text{Number of Accidents}}{\text{Number of Insureds}} = \frac{250}{200} = 1.250 \quad PAAF_{Overall} = \frac{250}{800 + 200} = .250$$

2) Calculate the past relative accident frequency:
$PRAF_{II} = PAAF_{II}/PAAF_{Overall} = 1.250/.250 = 5.000$

3) Calculate the future relative accident frequency. Assume that it varies directly with pure premium.

$$FRAF_{II} = \frac{\text{Future Pure Premium}_{II}}{\text{Future Pure Premium}_{Overall}} = \frac{300}{(.80)(250) + (.20)(300)} = 1.154$$

4) Calculate the credibility:

$$Z = \frac{FRAF_{II} - 1.0}{PRAF_{II} - 1.0} = \frac{1.154 - 1.000}{5.000 - 1.000} = .0385, \text{ pp. 159–60.}$$

A8. 1) Calculate the future relative claim frequency:

$$FRCF_5 = \frac{FACF_5}{FACF_{Overall}} = \frac{r/(a+n)}{r/a} = \frac{10/(100+5)}{10/100} = .952$$

2) Calculate the credibility: $Z_5 = 1 - FRCF_5 = 1 - .952 = .048$, pp. 159–60.

A9. 1) Calculate the past relative claim frequency:

$$PRCF_0 = \frac{PACF_0}{PACF_{Overall}} = \frac{m/(1 - e^{-m})}{m} = \frac{1}{1 - e^{-m}}$$

2) Calculate the future relative claim frequency:
$FRCF_0 = FRAF_0/FRAF_{Overall} = m(e^{-m} + 1)/m = e^{-m} + 1$

3) Calculate the credibility:

$$Z = \frac{FRCF_0 - 1.0}{PRCF_0 - 1.0} = \frac{e^{-m} + 1 - 1}{1/(1 - e^{-m}) - 1} = 1 - e^{-m}, \text{ pp. 159–60, 164.}$$

A10. Assume there are 10,000 risks with annual mean claim frequency of .05 and 10,000 risks with annual mean claim frequency of .15. Each risk's claim count distribution follows a Poisson process. Use the methods in the Bailey and Simon paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," and the data in the following table.

x	.05	.10	.15	.20	.25	.30
e^{-x}	.9512	.9048	.8607	.8187	.7788	.7408

Assume each risk's mean claim frequency remains the same from year to year. What is the credibility of the experience of a risk which has been claim-free for one year? For two years? (87–9–16a–3)

A11. The 1986 policy year collision experience of a sample of 100,000 cars, each of which had been insured for at least the preceding three years, was tabulated as follows:

Merit Rating Class Number of Years Claim-Free Prior to 1986 Policy Year	Policy Year 1986 Exposure (Car-Years)	Policy Year 1986 Number of Claims
3 or more	71,000	7,800
2	9,000	1,400
1	10,000	1,600
0	10,000	1,700
Total	100,000	12,500

Use the method of Bailey and Simon in their paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" to estimate the credibility of the experience of one car for one year. (88–9–11a–1)

A12. You are the actuary for the Hirate Insurance Company. Your indicated rate for drivers who have been accident-free for at least two years is $400 using the method described by Bailey and Simon in "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" and the following:

Years Claim-Free Prior to Policy Year 88	Earned Premium at Policy Year 88 Zero Years Claim-Free Rates	Policy Year 88 Number of Claims
3 or more	$400,000	1,200
2	80,000	720
1	80,000	1,000
0	80,000	1,716

What is the average rate for all drivers? (89–9–12–2)

A13. You are given the following information:

Group	Number of Years Claim-Free Before 1990	Earned Premium at 1990 Group C Rates	Prior Claim Frequency per $1,000 of Premium
A	2 or more	$300,000	
B	1	50,000	
C	0	150,000	
Total		$500,000	1.100

Using Bailey and Simon's techniques discussed in "An Actuarial Note on the Credibility of a Single Private Passenger Car," the credibility for two or more years claim-free is .20 and the credibility for one or more years claim-free is .1714. Calculate the number of claims observed for each group A, B, and C. (90–9-11–2)

A10. 1) Calculate the number of drivers in the one- and two-year claim-free categories, using the table, which represents the Poisson process:
$\text{Drivers}_1(.05) = (10{,}000)(.9512) = 9{,}512$ $\text{Drivers}_1(.15) = (10{,}000)(.8607) = 8{,}607$
$\text{Drivers}_2(.05) = (10{,}000)(.9512)^2 = 9{,}048$ $\text{Drivers}_2(.15) = (10{,}000)(.8607)^2 = 7{,}408$

2) Calculate the future absolute claim frequency for each category of driver as the weighted average of the mean claim frequencies:
$$\text{FACF}_1 = \frac{(9{,}512)(.05) + (8{,}607)(.15)}{18{,}119} = .0975$$
$$\text{FACF}_2 = \frac{(9{,}048)(.05) + (7{,}408)(.15)}{16{,}456} = .0950$$

3) Calculate future relative claim frequencies:
$\text{FRCF} = \text{FACF}/\text{FACF}_{\text{Overall}}$
$\text{FRCF}_1 = .0975/.1000 = .975$ $\text{FRCF}_2 = .0950/.1000 = .950$

4) Calculate credibilities:
$Z = 1 - \text{FRCF}$ $Z_1 = 1 - .975 = .025$ $Z_2 = 1 - .950 = .050$, pp. 159–60.

A11. 1) Calculate the future absolute claim frequencies:
$$\text{FACF} = \frac{\text{Number of Claims}}{\text{Car-Years}} \qquad \text{FACF}_1 = \frac{7{,}800 + 1{,}400 + 1{,}600}{71{,}000 + 9{,}000 + 10{,}000} = .120$$
$\text{FACF}_{\text{Overall}} = 12{,}500/100{,}000 = .125$

2) Calculate the future relative claim frequency:
$\text{FRCF}_1 = \text{FACF}_1/\text{FACF}_{\text{Overall}} = .120/.125 = .960$

3) Calculate the credibility:
$Z = 1 - \text{FRCF}_1 = 1 - .96 = .040$, pp. 159–60.

A12. 1) Calculate future absolute claim frequencies:
$$\text{FACF}_2 = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = \frac{1{,}200 + 720}{400{,}000 + 80{,}000} = .00400$$
$$\text{FACF}_{\text{Overall}} = \frac{1{,}200 + 720 + 1{,}000 + 1{,}716}{400{,}000 + 80{,}000 + 80{,}000 + 80{,}000} = .00724$$

2) Calculate the future relative claim frequency:
$\text{FRCF}_2 = \text{FACF}_2/\text{FACF}_{\text{Overall}} = .00400/.00724 = .552$

3) Calculate the credibility:
$Z_2 = 1 - \text{FRCF}_2 = 1 - .552 = .448$

4) Calculate the average premium:
$$\text{AP}_{\text{Overall}} = \frac{\text{Two-Year Claim-Free Premium}_2}{1 - Z_2} = \frac{400}{1 - .448} = 724.64, \text{ pp. 159–60.}$$

A13. 1) Calculate future relative claim frequencies:
$\text{FRCF} = 1 - Z$ $\text{FRCF}_A = 1 - .20 = .800$ $\text{FRCF}_{A+B} = 1 - .1714 = .829$

2) Calculate future absolute claim frequency for each group:
$\text{FACF} = (\text{FRCF})(\text{FACF}_{\text{Overall}})$
$\text{FACF}_A = (.800)(1.100) = .880$ $\text{FACF}_{A+B} = (.829)(1.100) = .912$

3) Calculate the claims for the three groups:
$\text{Claims} = (\text{FACF})(\text{Adjusted Earned Premium})/1{,}000$
$\text{Claims}_A = (.880)(300{,}000/1{,}000) = 264$ $\text{Claims}_{A+B} = (.912)(350{,}000/1{,}000) = 319$
$\text{Claims}_B = \text{Claims}_{A+B} - \text{Claims}_A = 319 - 264 = 55$
$\text{Claims}_C = \text{Claims}_{\text{Overall}} - \text{Claims}_{A+B} = (1.100)(500{,}000/1{,}000) - 319 = 231$, pp. 159–60.

A14. A simplified merit rating plan splits a class of insureds into three groups based entirely upon claim frequency as described in "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" by Bailey and Simon. Based upon the following chart and a standard rate of $1,500, what is the rate for an insured who has had no claims for at least one year?

Number of Years Since Last Accident	Earned Premiums at Present Rates	# of Claims
Two or More	$100,000	200
One	40,000	90
None	10,000	25

(91–9–27–2)

A15. Insureds in territory A experienced a claim frequency of .08 in 1991 and insureds statewide including territory A, experienced a claim frequency of .05 in 1991. In 1992 the formula relative claim frequency for territory A was 1.50. Based on Bailey and Simon's paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," what is the one-year credibility implied for risks in territory A?

A. < .200 B. ≥ .200 but < .400 C. ≥ .400 but < .600 D. ≥ .600 but < .800 E. ≥ .800 (93-9-6-1)

A16. Based on the methodology and notation used by Bailey and Simon in "An Actuarial Note on the Credibility of a Single Private Passenger Car" and the table below, calculate the credibility for category B risks (i.e., risks whose number of claim-free years equals zero) for a one-year experience period. (You can assume that the Poisson distribution reasonably approximates the distribution of observed claim counts among risks from all merit rating groups combined.) Show all of your work.

Merit Rating (Number of Accident-Free Years)	Earned Car-Years	Earned Premium at Present Category B Rates	Number of Claims Incurred
A (3+)	3,005,000	195,400,000	260,000
X (2)	148,000	10,700,000	18,000
Y (1)	184,000	13,200,000	25,000
B (0)	330,000	23,000,000	46,000
Total	3,667,000	242,300,000	349,000

(94–9–31–2)

A17. Based on Bailey and Simon's paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" and the information given below, calculate the credibilities that can be assigned to the experience of a single private passenger car from each of the following two groups: (Show all work.)

a. The group of risks that have been claim-free for two or more years
b. The group of risks that have been claim-free for no years.

Group	Number of Years Claim-Free	Earned Car Yearsa	Earned Premium t Present D Rates	Number of Claims Incurred
A	3 or more	650,000	390,000,000	54,250
B	2	200,000	120,000,000	21,000
C	1	75,000	45,000,000	10,125
D	0	75,000	45,000,000	14,625
Total		1,000,000	600,000,000	100,000

(95–9–30–1.5/1.5)

A14. 1) Calculate future absolute claim frequencies:

$$FACF_1 = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = \frac{200+90}{100{,}000+40{,}000} = .00207$$

$$FACF_{Overall} = \frac{200+90+25}{100{,}000+40{,}000+10{,}000} = .0210$$

2) Calculate the future relative claim frequency:
$FRCF_1 = FACF_1/FACF_{Overall} = .00207/.00210 = .986$

3) Calculate the credibility:
$Z_1 = 1 - FRCF_1 = 1 - .986 = .014$

4) Calculate the premium:
$Premium_1 = (Standard\ Premium_1)(1 - Z_1) = (1{,}500)(1 - .014) = 1{,}479$, pp. 159–60.

A15. 1) Calculate the past relative claim frequency.:
$PRCF_A = PACF_A/PACF_{Overall} = .08/.05 = 1.60$

2) Calculate the credibility: $Z_A = \frac{FRCF_A - 1.0}{PRCF_A - 1.0} = \frac{1.50 - 1.00}{1.60 - 1.00} = .833$, pp. 159–60.

Answer: E

A16. 1) Calculate future absolute claim frequencies:

$$FACF_0 = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = 46{,}000/23\bar{M} = .002$$

$FACF_{Overall} = 349{,}000/242.3\bar{M} = .00144$

2) Calculate the future relative claim frequency:
$FRCF_B = FACF_B/FACF_{Overall} = .002/.00144 = 1.389$

3) Calculate the past relative claim frequency: $m = .349\bar{M}/3.667\bar{M} = .095$
$PRCF_B = 1/(1 - e^{-m}) = 1/(1 - e^{-.095}) = 11.034$

4) Calculate the credibility: $Z = \frac{FRCF_B - 1.0}{PRCF_B - 1.0} = \frac{1.389 - 1.00}{11.034 - 1.00} = .039$, pp. 159–60, 164.

A17. a. 1) Calculate future absolute claim frequencies:

$$FACF_2 = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = \frac{54{,}250+21{,}000}{390\bar{M}+120\bar{M}} = .000148$$

$FACF_{Overall} = .1\bar{M}/600\bar{M} = .000167$

2) Calculate the future relative claim frequency:
$FRCF_2 = FACF_1/FACF_{Overall} = .000148/.000167 = .886$

3) Calculate the credibility:
$Z_2 = 1 - FRCF_2 = 1 - .886 = .114$

b. 1) Calculate the future absolute claim frequency:
$FACF_0 = 14{,}625/45\bar{M} = .000325$

2) Calculate the future relative claim frequency:
$FRCF_0 = FACF_0/FACF_{Overall} = .000325/.000167 = 1.946$

3) Calculate the past relative claim frequency:
$m = .1\bar{M}/1\bar{M} = .1$
$PRCF_0 = 1/(1 - e^{-m}) = 1/(1 - e^{-.1}) = 10.508$

4) Calculate the credibility:

$$Z = \frac{FRCF_0 - 1.0}{PRCF_0 - 1.0} = \frac{1.946 - 1.00}{10.508 - 1.00} = .099$$, pp. 159–60, 164.

A18. Based on Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," and using the information below, calculate the number of claims incurred for group C. Show all work.

Group	Number of Years Claim Free	Earned Car Years	Earned Premium at Present Group D Rates (000)	Number of Claims Incurred
A	3 or more	700,000	$420,000	62,376
B	2	175,000	105,000	15,957
C	1	100,000	60,000	??????
D	0	25,000	15,000	??????
Totals		1,000,000	$600,000	98,000

The credibility for the group of risks with one or more claim-free years (Z) equals .086. (98–9–26a–2)

A19. Based on Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" and the table below, answer the following.

Private Passenger Automobile Liability – Non-Farmers Class 3 – Business Use

Merit Rating	Earned Car Years	Earned Premium at Present B Rates	Number of Claims Incurred	Claim Frequency per $1,000 of Premium	Relative Claim Frequency
A	247,424	$25,846,000	31,964	1.237	0.920
X	15,868	1,783,000	2,695	1.511	1.123
Y	20,369	2,281,000	3,546	1.555	1.156
B	37,666	4,129,000	7,565	1.832	1.362
Total	321,327	34,039,000	45,770	1.345	1.000

where: Class A - three or more years claim-free
Class X - two years claim-free
Class Y - one year claim-free
Class B - zero years claim-free

Calculate the credibilities for a single private passenger car for one year, two years, and three years. Show all work. (00–9–32a–1.5)

A20. Use Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," and Hazam's discussion to answer the following questions. Using the information below on class 1, calculate the credibility for one-year and two-year claim-free periods for that class. Show all work.

	Number of Years Claim Free	Earned Premium at Present Rates	Number of Claims Incurred	Earned Car Years
Class 1	2 or more	$5,000,000	7,000	15,000
	1	$7,000,000	10,000	12,250
	0	$1,000,000	2,000	400
	Total	$13,000,000	19,000	27,650

(01–9–22a–2.5)

A18. 1) Calculate future absolute claim frequencies. Let X equal the number of claims incurred for risks who are claim-free for one year.

$$FACF_1 = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = \frac{62{,}376 + 15{,}957 + X}{420{,}000 + 105{,}000 + 60{,}000} = \frac{78{,}333 + X}{585{,}000}$$
$$FACF_{Overall} = 98{,}000/600{,}000 = .16333$$

2) Calculate the future relative claim frequency:

$$FRCF_1 = FACF_1/FACF_{Overall} = \frac{78{,}333 + X}{(585{,}000)(.16333)} = \frac{78{,}333 + X}{95{,}548}$$

3) Calculate X:

$$.086 = Z_2 = 1 - FRCF_1 = 1 - \frac{78{,}333 + X}{95{,}548} \qquad X = 8{,}998, \text{ pp. 159–60.}$$

A19. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$
$$FACF_1 = \frac{31{,}964 + 2{,}695 + 3{,}546}{25{,}846{,}000 + 1{,}783{,}000 + 2{,}281{,}000} = .001277$$
$$FACF_2 = \frac{31{,}964 + 2{,}695}{25{,}846{,}000 + 1{,}783{,}000} = .001254$$

2) Calculate the future relative claim frequencies:

$$FRCF_1 = FACF_1/FACF_{Overall} = .001277/.001345 = .949$$
$$FRCF_2 = FACF_2/FACF_{Overall} = .001254/.001345 = .932$$

3) Calculate the credibilities:

$$Z_1 = 1 - FRCF_1 = 1 - .949 = .051$$
$$Z_2 = 1 - .932 = .068 \qquad Z_3 = 1 - .920 = .080, \text{ pp. 159–60.}$$

A20. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} \qquad FACF_{Overall} = \frac{19{,}000}{13\bar{M}} = .001462$$
$$FACF_1 = \frac{10{,}000 + 7{,}000}{7\bar{M} + 5\bar{M}} = .001417 \qquad FACF_2 = \frac{7{,}000}{5\bar{M}} = .001400$$

2) Calculate the future relative claim frequencies:

$$FRCF_1 = FACF_1/FACF_{Overall} = .001417/.001462 = .969$$
$$FRCF_2 = FACF_2/FACF_{Overall} = .001400/.001462 = .958$$

3) Calculate the credibilities:

$$Z_1 = 1 - FRCF_1 = 1 - .969 = .031$$
$$Z_2 = 1 - .958 = .042, \text{ pp. 159–60.}$$

A21. Given the following data, calculate the credibilities for one-year and two-year claim-free periods. A represents three or more years since the most recent accident. X represents two years since the most recent accident. Y represents one year since the most recent accident. B represents zero years since the most recent accident.

	Earned Car Years	Earned Premium at Present Class B Rates	Number of Claims
A	50,000	$5,500,000	5,000
X	6,500	682,500	1,000
Y	5,000	535,000	850
B	4,500	490,500	900
Total	66,000	$7,208,000	7,750

(02–9–47a–1.5)

A22. You are given the following data:

Class	Years Since Last Accident	Actual Earned Premium at Present B Rates	Earned Car Years	Number of Claims
A	3+	375,000	2,500	200
X	2	15,000	100	12
Y	1	22,500	150	20
B	0	37,500	250	38

Assume that the same rate is charged to all insureds within a class and there have been no rate changes in or since the experience period.

a. What is the credibility of three or more accident-free years of experience?
b. What is the credibility of one or more accident-free years of experience? (03–9–22a&b–1ea.)

A23. Given the following information:

Class	Number of Years Since Most Recent Accident	Earned Car Years	Earned Premium at Present B Rates	Number of Claims
A	3 or more	10,000	$1,000,000	1,000
X	2	7,000	$770,000	1,155
Y	1	5,000	$625,000	1,250
B	0	2,000	$400,000	1,000
Total		24,000	$2,795,000	4,405

Calculate the credibility of one or more accident-free years of experience.

A. .087 B. .098 C. .153 D. .212 E. .257 (04–9–2–1)

A24. Given the following information:

N - number of drivers in the population
m - mean claim frequency for all drivers
Mod - credibility-weighted modification factor for risks with one or more claims in the past year

Derive the formula for the credibility assigned to the experience of drivers with one or more claims in the past year. Assume that claim frequency follows a Poisson distribution. (05–9–3a–2)

A21. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$

$$FACF_1 = \frac{5{,}000 + 1{,}000 + 850}{5{,}500{,}000 + 682{,}500 + 535{,}000} = .0010197$$

$$FACF_2 = \frac{5{,}000 + 1{,}000}{5{,}500{,}000 + 682{,}500} = .0009705 \quad FACF_{Overall} = \frac{7{,}750}{7{,}208{,}000} = .0010752$$

2) Calculate the future relative claim frequencies:

$FRCF_1 = FACF_1/FACF_{Overall} = .0010197/.0010752 = .948$

$FRCF_2 = FACF_2/FACF_{Overall} = .0009705/.0010752 = .903$

3) Calculate the credibilities:

$Z_1 = 1 - FRCF_1 = 1 - .948 = .052 \qquad Z_2 = 1 - .903 = .097$, pp. 159–60.

A22. a.&b. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$

$$FACF_3 = \frac{200}{375{,}000} = .0005333$$

$$FACF_1 = \frac{200 + 12 + 20}{375{,}000 + 15{,}000 + 22{,}500} = .0005624$$

$$FACF_0 = \frac{200 + 12 + 20 + 38}{375{,}000 + 15{,}000 + 22{,}500 + 37{,}500} = .0006$$

2) Calculate the future relative claim frequencies:

$FRCF_3 = FACF_3/FACF_{Overall} = .0005333/.0006 = .889$

$FRCF_1 = FACF_1/FACF_{Overall} = .0005624/.0006 = .937$

3) Calculate the credibilities:

$Z_3 = 1 - FRCF_3 = 1 - .889 = .111 \qquad Z_1 = 1 - .937 = .063$, pp. 159–60.

A23. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$

$$FACF_1 = \frac{1{,}000 + 1{,}155 + 1{,}250}{1{,}000{,}000 + 770{,}000 + 625{,}000} = .0014217$$

$$FACF_{Overall} = \frac{4{,}405}{2{,}795{,}000} = .0015760$$

2) Calculate the future relative claim frequencies:

$FRCF_1 = FACF_1/FACF_{Overall} = .0014217/.0015760 = .902$

3) Calculate the credibilities:

$Z_1 = 1 - FRCF_1 = 1 - .902 = .098$, pp. 159–60.

Answer: B

A24. $Mod = \frac{Z_0}{1 - e^{-m}} + (1 - Z_0) \qquad Z_0 = \frac{(Mod - 1)(1 - e^{-m})}{e^{-m}}$, pp. 159–160, 164.

A25. Given the following information about an automobile insurance portfolio:

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Claims Incurred
A	3	$25,000,000	40,000
B	2	8,000,000	15,000
C	1	13,000,000	25,000
D	0	8,000,000	30,000

Calculate the credibility of a single car for each of the following: one-year, two-year, and three-year accident-free periods. (06–9–2a–3)

A26. The following data were compiled from the ABC automobile insurance portfolio:

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Number of Claims Incurred
A	3 or more	$100,000,000	120,000
B	2	10,000,000	25,000
C	1	17,000,000	44,000
D	0	10,000,000	36,000

Calculate the credibility of a single car for each of the following ranges of accident-free years:

i) ≥ 1 ii) ≥ 2 iii) ≥ 3. (07–9–2a–1)

A27. A liability insurer collects the following data for a particular class of private passenger auto risks:

Accident-Free Years	Earned Exposures	Incurred Losses ($)
≥ 2	2,500	1,000,000
1	500	500,000
0	1,000	2,500,000
Total	4,000	4,000,000

Assume the following:

i) The base rate is $1,250 per exposure.
ii) An experience rating factor is the only factor applied to the base rate.

a. Calculate the credibility of an exposure that is accident-free for one or more years.
b. Calculate the premium for an exposure that is accident-free for two or more years. (08–9–5–1ea.)

A25. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$

$$FACF_1 = \frac{44,000 + 25,000 + 112,050}{17\bar{M} + 10\bar{M} + 100\bar{M}} = .001426$$

$$FACF_2 = \frac{25,000 + 112,050}{10\bar{M} + 100\bar{M}} = .001246$$

$$FACF_3 = \frac{112,050}{100\bar{M}} = .001125$$

$$FACF_{Overall} = \frac{36,000 + 44,000 + 25,000 + 112,050}{10\bar{M} + 17\bar{M} + 10\bar{M} + 100\bar{M}} = .001584$$

2) Calculate the future relative claim frequencies:
$FRCF_1 = FACF_1/FACF_{Overall} = .001426/.001584 = .900$
$FRCF_2 = FACF_2/FACF_{Overall} = .001246/.001584 = .787$
$FRCF_3 = FACF_3/FACF_{Overall} = .001125/.001584 = .710$

3) Calculate the credibilities:
$Z_1 = 1 - FRCF_1 = 1 - .854 = .146$ $Z_2 = 1 - .818 = .182$,
$Z_3 = 1 - .785 = .215$, pp. 159–60.

A26. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}} = \frac{44,000 + 25,000 + 120,000}{17\bar{M} + 10\bar{M} + 100\bar{M}} = .001488$$

$$FACF_2 = \frac{25,000 + 120,000}{10\bar{M} + 100\bar{M}} = .001318 \qquad FACF_3 = \frac{120,000}{100\bar{M}} = .001200$$

$$FACF_{Overall} = \frac{36,000 + 44,000 + 25,000 + 120,000}{10\bar{M} + 17\bar{M} + 10\bar{M} + 100\bar{M}} = .001642$$

2) Calculate the future relative claim frequencies:
$FRCF_1 = FACF_1/FACF_{Overall} = .001488/.001642 = .906$
$FRCF_2 = FACF_2/FACF_{Overall} = .001318/.001642 = .803$
$FRCF_3 = FACF_3/FACF_{Overall} = .001200/.001642 = .731$

3) Calculate the credibilities:
$Z_1 = 1 - FRCF_1 = 1 - .906 = .094$ $Z_2 = 1 - .803 = .197$,
$Z_3 = 1 - .731 = .269$, pp. 159–60.

A27. a. 1) Calculate future absolute pure premium: $PP = \frac{\text{Incurred Losses}}{\text{Earned Exposures}}$

$$PP_1 = \frac{.5\bar{M} + 1\bar{M}}{500 + 2,500} = 500$$

$$PP_{Overall} = \frac{2.5\bar{M} + .5\bar{M} + 1\bar{M}}{1,000 + 500 + 2,500} = 1,000$$

2) Calculate the future relative pure premium: $RPP_1 = PP_1/PP_{Overall} = 500/1,000 = .500$

3) Calculate the credibility: $Z_1 = 1 - RPP_1 = 1 - .500 = .500$, pp. 159–60.

b. $RPP_2 = PP_2/PP_{Overall} = 400/1,000 = .400$
$Z_2 = 1 - .400 = .600$
$M_2 = Z_2RPP_2 + 1 - Z_2 = (.600)(.400) + 1 - .600 = .640$
$Premium_2 = (\text{Base Rate})(M) = (1,250)(.640) = 800$, p. 160.

A28. The following information can be used to calculate the credibility assigned to the experience of a single private passenger car. Assume claim counts follow a Poisson distribution.

Group	Years Since Last Accident	Earned Car Years	Earned Premium at Present B Rates	Number of Claims
A	3 or more	650,000	400,000,000	50,000
X	2	230,000	150,000,000	20,000
Y	1	100,000	75,000,000	12,000
B	0	M	45,000,000	18,000
Total		980,000 + M	670,000,000	100,000

a. Calculate M, the earned car years for group B, given that the credibility for an insured that has had no claim-free years is equal to .167.
b. Calculate the credibility for the group of risks that have been claim-free for two or more years. (09–9–4–2.5/1)

A29. An insurance company has a private passenger auto book of business with the following claims experience. Calculate the credibility of a single car for a driver with one of more accident-free years.

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Number of Claims Incurred
A	3 or More	60,000,000	45,000
B	2	15,000,000	15,000
C	1	20,000,000	29,300
D	0	5,000,000	18,700
		100,000,000	108,000

(10–9–5–1)

A30. An insurance company is using a merit rating plan for drivers in two states. State X has the following claims experience:

Group	Number of Accident-Free Years	Earned Premium at Present Group D rates	Number of Claims Incurred
A	3 or more	$500,000	240
B	2	$150,000	125
C	1	$200,000	190
D	None	$300,000	300
Total		$1,150,000	855

State Y has the following relative claim frequencies for accident-free experience:

Number of Accident-Free Years	Relative Claim Frequencies to Total
3 or more	.70
2 or more	.77
1 or more	.84

Assuming that no new risks enter or leave either state, use relative credibility to explain which state has more variation in an individual insured's probability of an accident. (11–8–1–3)

A28. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$

$$FACF_0 = \frac{18{,}000}{45{,}000{,}000} = .0004$$

$$FACF_{Overall} = \frac{100{,}000}{670{,}000{,}000} = .00014925$$

2) Calculate the future relative claim frequencies:

$FRCF_0 = FACF_0/FACF_{Overall} = .0004/.00014925 = 2.68 = Mod$

3) Calculate the ratio of actual losses to expected losses:

$$R = \frac{Mod - 1 + Z}{Z} = \frac{2.68 - 1 + .167}{.167} = 11.06$$

4) Equate to the Poisson expression for R and solve for m:

$$R = 11.06 = \frac{1}{1 - e^{-m}} \qquad m = .0948$$

5) Equate to the actual frequency and solve for M;

$$m = .0948 = \frac{100{,}000}{980{,}000 + M} \qquad M = 74{,}852, \text{ pp. 159–160, 164.}$$

A29. 1) Calculate future absolute claim frequencies:

$$FACF = \frac{\text{Incurred Claims}}{\text{Earned Exposures}}$$

$$FACF_1 = \frac{29{,}300 + 15{,}000 + 45{,}000}{20\bar{M} + 15\bar{M} + 60\bar{M}} = .00094$$

$$FACF_{Overall} = \frac{108{,}000}{100\bar{M}} = .00108$$

2) Calculate the future relative claim frequency:

$FRCF_1 = FACF_1/FACF_{Overall} = .00094/.00108 = .870$

3) Calculate the credibility:

$Z_1 = 1 - FRCF_1 = 1 - .870 = .130$, pp. 159–60.

A30. 1) Calculate future absolute claim frequencies in state X:

$$FACF = \frac{\text{Number of Claims}}{\text{Earned Premium at Present Rates}}$$

$FACF_1 = (190 + 125 + 240)/(200 + 150 + 500)(1{,}000) = .000653$

$FACF_2 = (125 + 240)/(150 + 500)(1{,}000) = .000562$

$FACF_3 = 240/(500)(1{,}000) = .000480$

$FACF_0 = 855/1{,}150{,}000 = .000743$

2) Calculate the future relative claim frequencies in state X:

$FRCF_1 = FACF_1/FACF_{Overall} = .000653/.000743 = .879$

$FRCF_2 = FACF_2/FACF_{Overall} = .000561/.000743 = .755$

$FRCF_3 = FACF_3/FACF_{Overall} = .000480/.000743 = .646$

Since the credibilities, the complements of the relative claim frequencies, are greater in state Y, there is more variation in an individual insured's probability of an accident in that state.

A31. An actuary is evaluating a merit rating plan for private passenger cars. Given the following:

Number of Accident-Free Years	Earned Car Years	Number of Claims Incurred
2 or More	500,000	20,000
1	200,000	15,000
0	100,000	9,000
Total	800,000	44,000

- Frequency varies by territory.
- State law prohibits reflecting territory differences in rating.
- Annual claims for an individual driver follow a Poisson distribution.
- Claim cost distributions are similar across all drivers.

a. Identify one potential issue with the exposure base used. Briefly explain whether or not earned premium would be a better choice for the exposure base.

b. Calculate the credibility of one driver with one or more year's accident-free experience.

c. Calculate the credibility of one driver with 0 Accident-Free years.

(15-8-1-0.5/1.0/1.0)

A31. a. Using earned car years may create maldistribution because some territories (or other non- merit rating variables) may have higher frequency. But using premium assumes the high frequency is reflected in higher premium and territorial differentials are proper. However, state regulation prevents territorial rating. Therefore, territorial differentials are not proper, and premium is not necessarily a better exposure base.

b.

Number of Accident-Free Years	Earned Car Years	Number of Claims Incurred	Frequency
1 or More	700,000	35,000	0.050
C = 0	100,000	9,000	0.090
Total	**800,000**	**44,000**	0.055

Mod = Z * R + (1 – Z)

<u>For one or more year's accident-free:</u>

Mod = 0.05 / 0.055 = 0.909; R = 0;

$\Rightarrow 0.909 = 1 - Z$

$\Rightarrow Z = 0.091$

c. Current Average Claim Frequency = 0.055=(44,000 / 800,000)

Mod = Z * R + (1 – Z)

<u>Since prior claim experience follows Poisson distribution and average claim is non-zero:</u>

$\text{Mod} = 0.09 / 0.055 = 1.636$

$R = \frac{1}{(1-e^{-\lambda})}$, where λ = current average claim frequency = 0.055

$\Rightarrow 1.636 = 18.686 * Z + (1 - Z)$

$\Rightarrow Z = 0.036$

A32. An insurance company has a private passenger auto book of business *with* an experience modification factor in its rating plan.

Given the following:

- Annual claims for an individual driver follow a negative binomial distribution with r= 10.
- The expected claim frequency for the entire book of business is 0.101.
- The credibility for the group of risks that have had at least one accident in the last yearis 0.02.

a. Calculate the experience modification factor for a policy that has had at least one accident in the last year.

b. Describe why a class *with* a higher volume of claims and more exposures may have less credibility than a class with fewer claims and exposures

(19-8-3-1.25/0.5)

A32. a. z = 0.02 r = 10

$$.101 = \frac{10p}{1-p}$$

$$.101 - .101p = 10p$$

$$p = .01$$

$$\Pr(N=0) = \binom{9}{0}(1-.01)^{10}(.01)^{0}$$

$$\Pr(N=0) = .9044$$

$$R = \frac{1}{1-\Pr(N=0)}$$

$$R = \frac{1}{1-.9044} = 10.458$$

$$Mod = (.02)(10.458) + (1-.02)$$

$$Mod = 1.1892$$

b. Experience rating is meant to distinguish an individual within the class. If there is low variance within a class, then experience rating is not as useful, so credibility is lower, even if the class has high volume.

B. Other Topics

B1. What is the conclusion reached by Bailey and Simon in their paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car?" Also give the fundamental equation from which Bailey and Simon derived their conclusion. (69–9–6d–2.5)

B2. Bailey and Simon in their paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," use an unusual method of calculating claim frequencies. How was their expression for claim frequency different and why did they use it? (70–9–7b–3.3)

B3. Based on Bailey and Simon's discussions and assuming that any insured's chances for an accident remain constant from one year to the next and further, that no risks transfer from one class to another, estimate the approximate credibilities for at least two years and three years of accident-free driving, given the following:

Class	One-Year Credibility	Claim Frequency per $1,000 of Premium
1	.08	1.35
2	.08	1.50
3	.04	1.20
4	.04	1.10
5	.06	1.15
Total		1.30

(75S–9a–2b–4)

B4. Indicate whether statements below are true or false. Base your answers for on Bailey and Simon's paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car."

a. If the variation of individual insured's chances for an accident were the same in each class, the credibility (for experience rating) would be expected to vary approximately in proportion to the annual claim frequency.
b. Credibility for experience rating depends not only on the volume of the data in the experience period, but also on the mean loss cost of each class.
c. If an individual's chance for an accident remained constant from one year to the next and there were no new risks entering or leaving the class, then the credibilities for one, two and three years would vary approximately in proportion to the square root of the number of years. (86–9–8a-c–.5 ea.)

B5. In their paper, Bailey and Simon observe a different ratio of the two-year credibility to the one-year credibility than would result from the assumptions made in their model. Give three reasons that may explain the difference. (87–9–16b–1)

B6. You are the actuary for the XYZ Insurance Company. Currently, you are considering implementing an experience rating program for your private passenger automobile insureds based on each insured's experience. Your analysis shows that although an insured's past claim frequency is very credible in predicting future claim frequency, an insured's past loss ratio is not very credible in predicting the future loss ratio. Based on Bailey and Simon's paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," list two potential nonrandom causes of this phenomenon. (88–9–12–1)

B1. Bailey and Simon establish an equation for future frequency as a weighted average of the past experience for a subclass and that of the entire class. Since the resulting values for Z are significant, they find the process worthwhile, pp. 159–61.

B2. They use premium rather than car-years as an exposure base. "This avoids the maldistribution created by having higher claim frequency territories produce more X, Y, and Z risks and also produce higher territorial premiums," p. 159.

B3. 1) Determine the credibility constants:

$$K = \frac{n(1-Z)}{Z} = \frac{1-Z}{Z} \qquad K_{.08} = \frac{1-.08}{.08} = 11.5$$

$$K_{.04} = \frac{1-.04}{.04} = 24.0 \qquad K_{.06} = \frac{1-.06}{.06} = 15.7$$

2) Calculate the two- and three-year credibilities:

$Z = n/(n+K)$

Two-year: $Z_{.08} = 2/(2+11.5) = .148$

$Z_{.04} = 2/(2+24) = .077$

$Z_{.06} = 2/(2+15.7) = .113$

Three-year: $Z_{.08} = 3/(3+11.5) = .207$

$Z_{.04} = 3/(3+24) = .111$

$Z_{.06} = 3/(3+15.7) = .160$, p. 151.

B4. a. T, p. 160.

b. F, p. 160 – Substitute "the amount of variation of individual hazards within the class" for "the mean loss cost of each class."

c. F, p. 160 – Eliminate "the square root of."

B5. Credibility does not increase proportionately with the number of claim-free years for the following reasons:

1) An individual's accident propensity changes over time.
2) The population of a class changes as individuals enter and leave.
3) Individuals within a class have different accident propensities, which are markedly skewed.
4) In the credibility formula, Z is not exactly proportionate to n, pp. 160, 151.

B6. 1) The loss ratios reflect territorial and class differentials that may account for most of the individual variance.
2) The loss ratios also reflect variations in severity, which may be random, pp. 159, 161.

B7. State whether the following are true or false according to Bailey and Simon 's "An Actuarial Note on the Credibility of the Experience of Single Passenger Car." If a statement is false, briefly explain why.

a. Credibility for merit rating should be based only on the number of years in the experience period.
b. Relative claim frequency calculated on the basis of premium is used rather than claim frequency based on car-years to account for distributional problems caused by frequency differences between territories.
c. Individual risk credibility is smaller in a refined class rating scheme in which hazard does not vary significantly as compared to a class rating scheme in which hazard varies significantly within a class.
d. In class ratemaking, an increase in the volume of the experience would be expected to produce an increase in the reliability of the data in proportion to the increase in volume. (90–9–10–2)

B8. You are given the following private passenger auto results from state X. Answer the questions below based on Bailey and Simon's paper, "An Actuarial Note on the Credibility of a Single Private Passenger Car."

	1991 Claim Frequency	One–year Credibility	Three–year Credibility
Pleasure class A	.050	.10	.30
Business class B	.080	.12	.20

a. Which class has less within class variability in claim frequency? Explain your answer.
b. Which class has a more stable claim frequency over three years? Assume that no exposures enter or leave either class during the experience period and that the risk distribution in both classes is not markedly skewed. Explain your answer. (92–9–43–1)

B9. According to Bailey and Simon, in "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," which of the following statements are true?

1. For accident-free risks, the credibility is equal to 1.000 minus the modification.
2. One reason that relative credibilities calculated for two and three years of experience are lower than expected is that the chance of an accident for an individual can change during the year.
3. Experience rating is a procedure used to find the deviation of an individual risk from the average risk.

A. 1 B. 1,2 C. 1,3 D. 2,3 E. 1,2,3 (94–9–8–1)

B10. According to Bailey and Simon, in "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," which of the following statements are true?

1. Credibility for experience rating depends not only on the volume of data in the experience period, but also on the amount of variation of individual hazards within the class.
2. Credibility for experience rating should increase in proportion to the square root of the volume of data.
3. The fact that the relative credibilities calculated for two and three years of experience are lower than expected is partially caused by risks entering and leaving the class.

A. 1,2 B. 1,3 C. 2,3 D. 1,2,3 E. None of these answers are correct. (94–9–9–1)

B7. a. F, p. 160 – It also should reflect the variation of inherent hazards within a class.

b. T, p. 159.

c. T, p. 161.

d. F, p. 160 – It increases the reliability only in proportion to the square root of the volume.

B8. a. Compare the three-year credibility to the 1991 claim frequency. This is greater for class A (6) than for class B (2.5). This indicates that class B assigns relatively less credibility to the individual insured's experience and more to the class experience, which is more homogeneous, i.e., has less within-class variability, p. 160.

b. Compare the three-year credibility to the one-year credibility. This is greater for class A (3) than for class B (5/3). Since exposures remain the same and there is no marked skewness in the class distributions, the lower relative credibility for class B is attributable to changes in individual propensities for accidents and thus class A has a more stable class frequency, p. 160.

B9. 1. T, p. 159

2. T, p. 160

3. T, p. 160

Answer: E

B10. 1. T, p. 160

2. F, p. 160 – Substitute “class ratemaking” for “experience rating.”

3. T, p. 160

Answer: B

B11. According to Hazam's discussion of Bailey and Simon's paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," which of the following are true?

1. For a study like that presented by Bailey and Simon, the use of premium as a base is an improvement over the use of exposure as a base.
2. Using a premium base eliminates the maldistribution only if high-frequency territories are also high-premium territories or if territorial differentials are proper.
3. Bailey and Simon's statement "the credibilities for experience periods of one, two, and three years would be expected to vary approximately in proportion to the number of years" holds largely true only for low credibilities.

A. 1 B. 2 C. 1,3 D. 2,3 E. 1,2,3 (95–9–6–1)

B12. You have been retained as a consulting actuary for Hirisk Auto Insurance Company. The company has asked for you to determine if any of the three classifications in use is possibly in need of further refinement. The only data available are shown below:

	Claim Frequency per $1,000 Earned Premium
Class A total	1.625
Class B total	1.750
Class C total	2.212

Only Risks with 3 or More Years Loss Free	Earned Premium per Earned Car Year	Credibility of a Single Risk
Class A	$150	.082
Class B	148	.046
Class C	190	.079

Using the procedures and formulas from Bailey and Simon's paper, "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," determine whether one or more of the current classes exhibits more variation of individual hazards than does the others. Assume that the earned premiums are adjusted to a common current rate level. Show all work. (95–9–32–3)

B13. You are given the following private passenger automobile results for a hypothetical state. Using the techniques from Bailey and Simon's "An Actuarial Note on the Credibility of a Single Private Passenger Car," answer the questions below:

Class	Description
A	Pleasure class – unmarried male operator under age 25
B	Pleasure class – unmarried female operator under age 25
C	Pleasure class – operator over age 55

Class	1995 Claim Frequency	1995 One-Year Credibility	1993-1995 Three–Year Credibility
A	.12	.18	.36
B	.10	.08	.22
C	.08	.16	.48

a. Which class has a more stable claim frequency over the three-year period? Assume that there is no change in the exposures in each class during the three years and that the risk distribution in each class is not markedly skewed. Explain your answer.

b. Which class has less variability in claim frequency within its class? Explain your answer. (96–9–50–1/1)

B11. 1. T, p. 165

2. F, p. 165 – Substitute "and" for "or."

3. T, p. 165

Answer: C

B12. 1) Calculate total claim frequencies per car-year:

TCF = (Claim Frequency per \$1,000)(Earned Premium)/1,000
TCF_A = (1.625)(150)/1,000 = .244
TCF_B = (1.750)(148)/1,000 = .259
TCF_C = (2.212)(190)/1,000 = .420

2) Calculate claim frequencies per car-year for risks loss-free for three or more years:

CF = (1 – Z)TCF
CF_A = (1 – .082)(.244) = .224
CF_B = (1 – .046)(.259) = .247
CF_C = (1 – .079)(.420) = .387

3) Calculate the ratios of credibility to claim frequency:

Ratio = Z/CF
$Ratio_A$ = .082/.224 = .366
$Ratio_B$ = .046/.247 = .186
$Ratio_C$ = .079/.387 = .204

Since class A has the highest ratio, it assigns the most credibility to individual insureds' experience and the least to class experience, which is less homogeneous. It thus has more within-class variability. Since class C has the second highest ratio, it has the second greatest variability, pp. 160, 163.

B13. a. Compare the three-year credibility to the one-year credibility. This is greater for class C (3) than for class A (2) or class B (2.75). Since exposures remain the same and there is no marked skewness in the class distributions, the lower relative credibilities for classes A and B are attributable to changes in individual propensities for accidents and thus class C has a more stable class frequency, p. 160.

b. Compare the three-year credibility to the 1995 claim frequency. This is greater for class C (6) than for class A (3) or class B (2.2). This indicates that class B assigns relatively less credibility to individual insureds' experience and more to the class experience, which is more homogeneous, i.e., has less within-class variability, pp. 160, 163.

B14. According to Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," which of the following are true?

1. Relative claim frequency is calculated on a premium basis to avoid biases due to the fact that exposure-based frequency varies by territory.
2. Credibility for experience rating depends only on the volume of data in the experience period.
3. The experience for one car for one year has significant and measurable credibility for experience rating.

A. 3 B. 1,2 C. 1,3 D. 2,3 E. 1,2,3 (97–9–19–1)

B15. Based on Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car, answer the following questions:

a. What conclusions do the authors reach with respect to merit rating using one year's worth of experience?
b. In a highly refined private passenger rating classification system, what relative credibilities would the authors conclude should be assigned to the experience of an individual risk to the experience of a class? (98–9–26b&c–.5/.5)

B16. In Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," the authors state that under certain conditions, the credibilities associated with experience periods of one, two, and three accident-free years for insureds within a given class would be expected to vary approximately in proportion to the number of years. Which of the following are reasons why this would not be true?

1. Changes in an individual insured's chance for an accident within a year
2. Skewness in the risk distribution of individual insureds
3. The impact of risks entering and leaving the class

A. 1 B. 1,2 C. 1,3 D. 2,3 E. 1,2,3 (99–9–1–1)

B17. a. Briefly describe the relationship that Bailey and Simon expect between the three credibilities calculated in A19.
b. Do the credibilities calculated in A19. follow the relationship described in a.? Briefly explain why or why not. (00–9–32b&c–.5/1)

B18. According to Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," which of the following is false?

A. The experience for one car for one year has significant and measurable credibility for experience rating.
B. Credibility for experience rating depends on the variation of individual hazards within the class.
C. In a highly refined private passenger rating classification system that reflects inherent hazard, there would not be much accuracy in an individual risk merit rating plan.
D. In experience rating, an increase in the volume of data in the experience period increases the reliability of the indication in proportion to the square root of the volume.
E. None of these statements are false. (01–9–2–1)

B19. a. What exposure base do the authors use? Explain why.
b. According to Hazam, what two conditions must be met to use the exposure base described in a.? (01–9–22b&c-.5/.5)

B14. 1. T, p. 159

2. F, p. 160 – It also reflects the variation of individual hazards within a class.

3. T, p. 161

Answer: C

B15. a. It "has significant and measurable credibility for experience rating."

b. In a highly refined system with homogenous classes, little weight should be assigned to the experience of an individual risk relative to that of the class, p. 161.

B16. E, p. 160.

B17. a. The credibility should "vary approximately in proportion to the number of years," p. 160.

b. See B5. No. The credibility for two accident-free years is less than two times that of one accident-free year; similarly, the credibility for three accident-free years is less than three times that of one accident-free year.

B18. A. T, p. 161

B. T, p. 160

C. T, p. 161

D. F, p. 160 – Substitute "class ratemaking" for "experience rating."

Answer: D

B19. a. See B2.

b. 1) "[H]igh frequency territories are also high premium territories."
2) "[T]erritorial differentials are proper," p. 151.

B20. See A21. Give two possible reasons that the two-year credibility is less than two times the one-year credibility. (02–9–47b–.5)

B21. Which of the following statements is false for private passenger auto experience rating?

A. Credibility assigned to an individual risk within a highly refined classification rating plan would be higher than the credibility assigned in a less-refined rating plan.
B. Credibility for experience rating depends on the amount of variation of individual hazard within the class.
C. Credibility for experience rating is significant and measurable when based on data from one car for one year.
D. Credibility for classification rating increases in proportion to the square root of the volume of data.
E. Credibility within a highly refined private passenger classification rating system would be larger where a wide range of hazard is encompassed within a classification. (03–9–2–1)

B22. See A22. Give two possible reasons why the answer in a. is not three times the answer in b. (03–9–22c–1)

B23. If there is a switch from a less-refined class plan to a highly refined class plan, describe the likely change in the credibility assigned to an individual risk. (05–8–3b–1)

B24. In performing credibility calculations, would using car-years instead of earned premium as an exposure base be more preferable? Explain why or why not. (06–9–2b–1)

B25. a. See A26. The following table provides the single-car credibility for the XYZ automobile insurance portfolio:

Accident-free years	≥ 1	≥ 2	≥ 3
Single-car credibility	.14	.10	.06

Discuss two conclusions that can be drawn from the different credibility results of the ABC and XYZ portfolios.

b. Explain why analysis of two portfolios with different classification plans could assign different values to the credibility of the experience of a single car. (07–9–2b&c–1/.5)

B20. See B5.

B21. A. F, p. 161 – Substitute “lower” for “higher.”

B. T, p. 160

C. T, p. 161

D. T, p. 160

E. T, p. 161

Answer: A

B22. See B5.

B23. As a class plan becomes more refined, the credibility assigned to an individual risk declines, p. 161.

B24. See B2.

B25. a. 1) Since XYZ's plan assigns less credibility to the experience of individual cars, its class plan may be more refined.

2) Since the credibilities in XYZ's plan are inversely related to the number of years without an accident, its portfolio of cars may be less stable, i.e., it has a lower retention rate, pp. 160–61.

b. A more-refined class plan will assign less credibility to the experience of an individual car and more to the experience of the class, p. 160.

B26. An insurance company has a private passenger auto book of business with the following claims experience:

Territory	Years Since Last Accident	Earned Premium at Present Rates for Two Years Since Last Accident	Earned Car Years	Number of Claims	Incurred Loss
1	0	$15,000,000	15,000	5,000	$9,000,000
1	1	$125,000,000	125,000	41,000	$75,000,000
1	2+	$230,000,000	230,000	76,000	$138,000,000
2	0	$25,000,000	25,000	7,000	$16,000,000
2	1	$310,000,000	300,000	84,000	$187,000,000
2	2+	$550,000,000	535,000	147,000	$328,000,000
3	0	$10,000,000	10,000	4,000	$7,000,000
3	1	$80,000,000	100,000	35,000	$43,000,000
3	2+	$160,000,000	170,000	60,000	$100,000,000

Choose an appropriate exposure base for calculating credibility. Justify the selection. (12–8–6–2.5)

B26. Premium should be used as the base to prevent the maldistribution of premium IF higher frequency territories have higher premiums AND territory differentials are correct.

Terr	(A) EP	(B) ECY	(C) # Clms	(D) Losses	(A)/(B) Avg EP	**Relative EP**
1	370,000,000	370,000	122,000	222,000,000	1,000	**1.0033**
2	885,000,000	860,000	238,000	531,000,000	1,029	**1.0325**
3	250,000,000	280,000	99,000	150,000,000	893	**0.8958**
Total	1,505,000,000	1,510,000	459,000	903,000,000	997	**1.0000**

Terr	(C)/(D) Avg PP	Rel PP	(C)/(B) Freq	**Relative Freq**
1	1,820	0.9249	0.3297	**1.0847**
2	2,231	0.1341	0.2767	**0.9104**
3	1,515	0.7702	0.3536	**1.1632**
Total	1,967		0.3040	**1.000**

Compare Relative Frequency to Relative EP. They are not lining up. Territory 3 has the highest relative frequency but the lowest relative EP. That means Earned Car Years is a more appropriate base than Earned Premium.

B27. The following data shows the experience of a merit rating plan for a specific state.

Number of Accident-Years	Earned Car Years	Earned Premium ($000)	Number of Incurred Claims
3 or More	250,000	250,000	1,200
2	300,000	100,000	625
1	25,000	100,000	750
0	12,000	150,000	1,500
Total	587,000	600,000	4,075

The base rate is $1,000 per exposure. No other rating variables are applicable.

a. The typical exposure base used to develop the merit rating plan is earned premium. Briefly discuss two assumptions in selecting this exposure base.

b. Calculate the ratio of credibility for an exposure with two or more years accident-free experience to one or more years accident-free experience.

c. Calculate the premium for an exposure that is accident free for two or more years.

(14-8-5-0.5/1.5/0.5)

B27. a. The 2 assumptions made are:

1. High frequency territories are also high premium territories
2. Territorial differentials are proper / adequate

b. Frequencies:

- Freq 2 or more = (1,200+625)/(250,000+100,000) = 0.0052
- Freq 1 or more = (1,200+625+750)/(250,000+100,000+100,000) = 0.0057
- Freq total = 4,075/600,000 = 0.0068

Mod factor:

- Mod 2 or more = 0.0052/0.0068 = 0.7677
- Mod 1 or more = 0.0057/0.0068 = 0.8425

Credibility factors:

- Cred 2 or more = 1 – Mod 2 or more = 1 – 0.7677 = 0.2323
- Cred 1 or more = 1 – Mod 1 or more = 1 – 0.8425 = 0.1575

Ratio = 0.2323/0.1575 = 1.4750

c. Premium = Base Rate x Mod Premium = 1000 x 0.7677 = $ 767.7

B28. A group of insureds have different expected claim frequencies. The number of insureds claim-free for the past *t* years is as follows:

Expected Claim Frequency	t=0	t=1	t=2	t=3
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Determine whether the variation of an individual insured's chance for an accident changes over time.

(16-8-1-2.75)

B28.

	(1)	(2)	(3)	(4)	(5)
n	# Claim free n or more years	Expected Claims	Frequency	Relative Frequency	Z
3	94,000	8,600	0.0915	0.9525	0.0475
2	198,500	18,450	0.0929	0.9677	0.0323
1	311,500	29,425	0.0945	0.9835	0.0165
Total	436,500	41,925	0.0960	1	

Expected claims:

t=3: 44,000 x 0.05 + 36,000 x 0.10 + 14,000 x 0.20 = 8,600

t=2: 45,000 x 0.05 + 43,000 x 0.10 + 16,500 x 0.20 = 9,850

t=1: 47,500 x 0.05 + 45,000 x 0.10 + 20,500 x 0.20 = 10,975

Total: 186,500 x 0.05 + 174,000 x 0.10 + 76,000 x 0.20 = 41,925

(3) = (2)/(1)
(4) = (3)/((3) Total)
(5) = 1 – (4)

If the variation of an insured's chance for an accident is not changing over time, then the 3-year credibility/1-year credibility will be approximately equal to 3 and the 2-year credibility/1-year credibility will be approximately equal to 2.

3+ year Z / 1+ year Z = 0.0475 / 0.0165 = 2.88
2+ year Z / 1+ year Z = 0.0323 / 0.0165 = 1.96

The ratios are approximately 3 and 2; the chance for accident is stable.

This problem also can be approached using the correlation test from Mahler's "An Example of Credibility and Shifting Risk Parameters." This produces a different conclusion. See Mahler problem 22.

B29. The following data shows the experience of a merit rating plan for private passenger vehicles. The merit rating plan uses multiple rating variables, including territory.

Number of Accident-Free Years	Earned Car Years (000s)	Earned Premium ($000s)	Number of Incurred Claims
5 or More	250	500,000	15,000
3 and 4	100	90,000	13,500
1 and 2	80	60,000	8,000
0	70	50,000	10,500
Total	500	700,000	47,000

Territory	Frequency	Average Premium
A	0.05	1,500
B	0.10	2,000
C	0.15	1,250

a. Recommend and justify an exposure base for this merit rating plan.

b. Calculate the relative credibility of an exposure that has been three or more years accident-free using the exposure base from part (a) above.

(17-8-3-0.75/0.75)

B29. a. Need one of these:

Earned Car Years: Using earned premium as the exposure base only corrects the maldistribution due to correlation between frequency & territory when:

1. Territory differentials are proper, and
2. High frequency territories are also high premium territories

Here Territory C has highest frequency but not highest premium, so #2 not satisfied.

Hence, using earned premium as base does not make an improvement. Therefore use earned car years as the exposure base.

Earned Premium: Territory is a variable that tends to be correlated with other risk characteristics, so it would be advisable to use earned premium as an exposure base to correct for exposure correlation, but only if high frequency territories also have high average premium.

This doesn't seem to be the case (i.e. Territory C is highest frequency, but lowest average premium) but premium could reflect other variables' impact, so use Earned Premium as exposure base.

b. If using Earned Car Years:

3+: (13,500+15,000)/(100+250) = 81.43
Total: 47,000/500 = 94
Rel Freq: 81.43/94 = 0.866
Z = 1-0.866 = 0.134

1+: (13,500+15,000+8,000)/(250+100+80) = 84.88
Rel Feq: 84.88/94 = 0.903
Z = 1-0.903 = 0.097

Relative credibility = 0.134/0.097 = 1.38

If using Earned Premium:

3 or more years claim frequency: (13,500+15,000)/(500,000+90,000) = 0.048
Total claim frequency: 47,000/700,000 = 0.067
Relative claim Frequency of 3 or more years: 0.048/0.067 = 0.72
Z = 1-0.72 = 0.28

1 or more years claims frequency: (13,500+15,000+8,000)/(500,000+90,000+60,000) = 0.056
Relative claim frequency of 1 or more years = 0.056/0.067 = 0.84
Z = 1-0.84 = 0.16

Relative credibility = 0.28/0.16 = 1.75

B30. An insurance company has a private passenger auto book of business with the following claims experience:

Group	Number of Accident-Free Years	Earned Premiums	Current Merit Rating Factor	Number of Claims Incurred
A	3 or more	216,000,000	0.60	25,000
X	2	135,000,000	0.75	18,000
Y	1	63,750,000	0.85	20,000
B	0	200,000,000	1.00	C
Total		614,750,000		63,000 + C

- Claim counts follow a Poisson distribution with parameter $\lambda = 0.05$.
- The credibility for the new policy period for an insured that has had no claim-free years is equal to 0.038.

b. Calculate C, the number of claims incurred for Group B.

c. Calculate the merit rating factor for an exposure that is accident-free for two or more years for the new policy period.

d. Briefly explain two circumstances under which using earned premium as the exposure base would not correct for maldistribution.

(18-8-3-1.5/0.75/0.5)

B30. a. $Mod = ZR + (1 - Z)$

$$R = \frac{1}{1 - e^{-\lambda}} = \frac{1}{1 - e^{-0.05}} = 20.504$$

Z=0.038

Mod = 0.038(20.504) + (1-0.038) = 1.7411

Assume earned premium not at present B rates. Adjusted earned premiums:

$$\text{A} = \frac{216,000,000}{0.6} = 360,000,000$$

$$\text{X} = \frac{135,000,000}{0.75} = 180,000,000$$

$$\text{Y} = \frac{63,750,000}{0.85} = 75,000,000$$

$$\text{B} = \frac{200,000,000}{1.0} = 200,000,000$$

Total = 815,000,000

$$\frac{\frac{\text{C}}{200,000,000}}{\frac{63,000 + \text{C}}{815,000,000}} = 1.7411$$

$$\frac{\text{C}}{200,000,000} = \frac{109,689.3 + 1.7411\text{C}}{815,000,000}$$

$$2.19 \times 10^{13} + 348,220,000\text{C} = 815,000,000$$

C=47,000

b. Relative frequency = mod

$$\text{Mod for A+X} = \frac{\frac{25,000 + 18,000}{360\text{M} + 180\text{M}}}{\frac{109,948}{815\text{M}}} = 0.59$$

Merit factor is 0.59.

c. 1) When high claim frequency territories are not high average premium territories
2) When territorial differentials are not proper